E 1088 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (CBCSS) EXAMINATION, NOVEMBER 2010

First Semester

Core Course—FOUNDATION OF MATHEMATICS

[Common for B.Sc. Mathematics (Model – I)
B.Sc. Mathematics Model – II and B.Sc. Computer Applications (Three Main)]

Time : Three Hours Maximum weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
A bunch of **four** *questions has weight* 1.

I. 1 What is the cardinality of the set, $\{\phi, \{a\}, a, \{a, \{a\}\}\}$.

2 What is $\bigcap_{i=1}^{\infty} A_i$ if $Ai = \{1; 2, 3, \ldots, i\}$?

3 Give an example of two one-to-one onto functions f and g form $\mathbb{R}$ to $\mathbb{R}$, but $f + g$ is neither one-to-one nor onto.

4 Give an example of a relation on the set of positive integers which is not symmetric but antisymmetric.

II. 5 Write a relation on the set $A = \{a, b, c, d\}$.

6 Let R_1 be the "less than" relation on the set of real numbers and let R_2 be the "greater than" relation on the set of real numbers. What is $R_1 - R_2$?

7 List the triples in the relation $\{(a, b, c) / a, b,$ and c are integers with $0 < a < b < c < 5\}$.

8 Give an example of a poset which is not totally ordered.

III. 9 Write the inverse of the following conditional statement "I go to the beach whenever it is a sunny summer day".

10 What is the truth value of $\forall x P(x)$, when $P(x)$ is the statement "$x^2 < 10$" and the domain consists of positive integers.

11 What is $x \oplus y$ if $x = 1, y = 0$.

12 Let p, q, r be the propositions :

p : You have the flu

q : You miss the final examination

r : You pass the course

Express the proposition $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ as an English sentence.

Turn over

IV. 13 Find the number of integers less than n and prime to n when $n = 1026$.

14 If $d_1, d_2, \ldots, d_r$ are the divisors of 24, what is $\phi(d_1) + \phi(d_2) + \ldots + \phi(d_r)$?

15 Find a number x such that $2x \equiv 1 (\text{mod } 5)$.

16 What are the roots of the congruence $x^4 - 1 \equiv 0 \pmod 5$?

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each questions has weight 1.

17 What are the terms a_0, a_1, a_2, a_3 of the $\{a_n\}$, where $a_n = 2^n + (-2)^n$.

18 What is the composite of the relations R and S, where R is the relation from {1, 2, 3} to {1, 2, 3,4} with R = {(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)} and S is the relation from {1, 2, 3,4} to {0, 1, 2) with S = {(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)} ?

19 Represent the relation {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3,4)} on { 1, 2, 3, 4) with a matrix.

20 Use De Morgan's laws to find the negation of the statement " Jan is rich and happy".

21 Show that the hypotheses $(p \wedge q) \vee r$ and $r \rightarrow s$ imply the conclusion $p \vee s$.

22 Prove that if r is the remainder when a is divided by b, the common divisors of a and b are the same as those of b and r.

23 Solve : $15x \equiv 6 \pmod{21}$.

24 Prove that the fifth power of any number N has the same right hand digit as N.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Prove that $\overline{A \cap B} = \bar{A} \cup \bar{B}$, where $\bar{A}$ denotes the complement of A with respect to the universal set u.

26 What are the equivalence classes of 0 and 1 for congruence module 4 ?

27 Give a proof by contradiction of the theorem "If $3n + 2$ is odd, then n is odd".

28 Translate the statement "The sum of two positive integers is always positive" into a logical expression.

29 Prove that the sequence of primes is endless.

30 Prove that every composite number has at least one prime divisor.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 Determine whether the relation represented by the matrix $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ is a partial order.

32 Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

33 Prove that if p is a prime number, then $\lfloor p-1+1$ is divisible by p.

$(2 \times 4 = 8)$